whence a graphical determination of the values of ω is easily inferred (pp. 50, 51, footnote).

Another interesting paper due to Euler is "De altitudine columnarum sub proprio pondere corruentium" (1778), investigating the height at which a mast or tree will begin to bend under its own weight. To this paper he might well have prefixed the old German proverb, quoted by Goethe in "Wahrheit und Dichtung":—"Es ist dafür gesorgt, dass die Baüme nicht in dem Himmel wachsen." We know now that the functions of Bessel are required for the complete analytical solution of this question, though the *Theorema maxime memorabile* enunciated by Euler, "Maxima altitudo, qua columnæ cylindricæ ex eadem materia confectæ, proprium pondus etiamnunc sustinere valent, tenet rationem subtriplicatam amplitudinis," is interesting as one of the first applications of the principle of mechanical similitude, showing why the proportions of the giant of the forest are stunted compared with those of the young tree, and also why it is hopeless to attempt the problem of human flight while g is 32.

Lagrange considered the same subject in "Sur la figure des colonnes" (1770), examining and disproving the dictum of Vitruvius that the *renflement* of a column was necessary for strength: the dictum can hardly be called an architectural fallacy, as the *renflement* corrects the tendency, due to *irradiation*, of a perfectly cylindrical column to appear attenuated in the middle; for a similar reason it is necessary to slightly blunt the neighbourhood of the point of a Gothic spire to avoid the appearance of concavity.

Coulomb, a well-known name to electricians, is mentioned by Saint-Venant as giving about this time (1780), in "Remarques sur la rupture des corps," the true position of the neutral line of a beam, although it is asserted by Dr. Todhunter that the ancient erroneous idea prevailed into the present century.

In Chapter II. the work of Young, Gregory, Eytelwein, Plana, Dupin, Belli, Binet, Biot, Rennie, Barlow, Tredgold, Fourier, Nobili, Bordoni, Hodgkinson, and others is analysed. Of these the English writers, who generally were experimentalists as well as theorists, are severely handled by Dr. Todhunter for their heresies on the neutral axis. Considering that the neutral axis is a mathematical fiction, depending on an ignoration of the shearing stress, and the consequent warping of the normal sections of a beam, this treatment of Dr. Todhunter is too severe, compared with the leniency with which he views the metaphysical speculations of the pure theorists. These experimentalists were trusted in their advice on important constructions, and took care their formulæ erred on the right side of strength.

To Navier (1821) we are first indebted for the general mathematical equations of the equilibrium and vibrations of an elastic solid, to be satisfied in the interior and at the surface, and henceforth the researches of mathematicians take a bolder flight from the treatment of the simple beam of former investigators.

Mlle. Sophie Germain's "Recherches sur la théorie des surfaces élastiques" (1821) appears to afford Dr. Todhunter gratification in showing that sex can make itself apparent even in mathematics. However, it is dangerous to argue from this instance, as hardly any mathematician has yet written on elastic surfaces without falling into error in the boundary conditions, and the subject is even now not yet certainly settled.

The vibration of elastic surfaces is important in its bearing on acoustics and music, and received about this time experimental and theoretical treatment from Chladni, Strehlke, Pagani, and Savart.

Chapters IV. and V. give an account of the treatment of the subject by the celebrated mathematicians Poisson and Cauchy, who practically exhausted the soluble problems, if we except the torsion questions considered by Saint-Venant. Poisson's results are generally expressed by means of definite integrals, most of which we see now can be classified as Bessel's functions. Both Poisson and Cauchy appear to have considered the subject of elasticity principally in its bearing on the new theory of physical optics, then receiving such important experimental and theoretical treatment at the hands of Fresnel.

Henceforth the theory receives development at the hands of so many writers that it is possible only to specify the honoured names of Gerstner, Green, McCullagh, Poncelet, and Maxwell as having contributed important advance to the subject.

Lamé's "Theory of Elasticity," carefully analysed in Chapter VII., still remains a standard text-book, in conjunction with the treatises of F. Neumann and Clebsch.

The volume concludes with an account of Saint-Venant's researches before 1850, the subsequent work to be recorded in the second volume. Saint-Venant is the name most honoured by practical elasticians and engineers, inasmuch as he has developed his theories from the definite practical problems presented by the large and daring constructions in iron and steel which mark the middle of this century.

In the appendix Mr. Pearson has carefully analysed the conflicting notations of different writers, and proposed a very convenient terminology and notation, which would save great trouble if universally adopted. He has also given an account of experiments carried out by Prof. Kennedy in his mechanical laboratory, which have an important bearing on the limitations of the truth of Hooke's law, or, in the language of elasticity, the constancy of the ratio of stress to corresponding strain.

The present volume is an indispensable hand-book of reference for the mathematician and the engineer, and in the editing and printing must be considered a very fitting tribute to the wonderful industry and application of its projector, the late Dr. Todhunter.

A. G. GREENHILL

THE ENCYCLOPÆDIA BRITANNICA

The Encyclopædia Britannica. Vol. XX. Pru—Ros. Vol. XXI. Rot—Sia. (Edinburgh: A. and C. Black, 1886.)

THE leading scientific articles in these two volumes are mainly biological. In Vol. XX. Prof. A. Newton contributes the articles on the various important groups of birds; and in those on the Quail, Screamer, Secretary Bird, Seriema or Cariama, it is truly surprising to find so many facts condensed into so small a compass. Mr. C. T. Newton's article on Pterodactyles gives us the newest information on this strange group of fossil reptiles. In the article on Reproduction only the broadest aspects of the phenomena attending it are glanced at, Mr. P. Geddes treating of the Animal, and Mr. S. H.

Vines of the Vegetable, group ; though, as was to have been expected, the early phenomena in the two kingdoms are to some extent the same. The facts mentioned are well up to date, and both portions of this article are worthy of their authors. Animal reproduction is illustrated by a few useful woodcuts ; but this help to the understanding of the text is wanting in the section on vegetable reproduction. Mr. Vines very correctly criticises Strasburger's idea that the cause of sexuality in cells is purely quantitative. In the article on Reptiles Dr. Günther finds himself on familiar ground, and in the forty pages placed at his disposal he gives a most excellent account of the history and literature of the group, and of the evolution of their classification, from Linnæus to Cope, followed by a brief record of some of the more important systematic works on reptiles, in which the need of a general work on the subject is pointed out. The principal faunistic works are alluded to, and then follow the systematic portions, the anatomy of the chief forms characteristic of the orders, and a paragraph about the distribution of reptiles in time and space. Prof. A. Gamgee, in an article on Respiration, dwells chiefly on the phenomena attending this function in mammals, the phenomena to be observed in all the other groups of the animal kingdom being scarcely even alluded to. The articles on the Rhinoceros and the Seal, by Mr. Flower, are quite models of encyclopædic articles, for in them we have just the information a general reader would require, and this of a thoroughly trustworthy kind. The same may be said of an article on Rhubarb, by Mr. E. W. Holmes.

The article on Rotifera, in Vol. XXI., by Prof. A. G. Bourne, was probably printed ere the finely illustrated monograph of this group by Gosse and Hudson had made its appearance, but the classification given is based on that of Hudson. The account of the general morphology and anatomy is well done, and in a few very pertinent remarks on their affinities the author concludes that, while the high development of the mastax, the specialised character of the lorica in many forms, the movable spines in Polyarthra, the limbs of Pedaliòn, and the lateral appendages in Asplanchna, the existence of a diminutive male, the formation of two varieties of ova—all point to a specialisation in the direction of the groups of the Mollusca, Arthropoda, and Chætopoda ; yet such phenomena would not justify the definite association of the Rotifera in a single phylum with any of them. The phenomena of rotifers being desiccated, and then coming into active existence, are mentioned as if actually proved by exact experiment ; but is this so? It certainly does not always succeed, as, no doubt, numerous observers have often noted : too frequently, from inattention, all the water will evaporate from a slide with rotifers ; and so far, general experience proves, that if this evaporation be carried to desiccation, not all the drops of water in the world will set up the rotifers that were on such a slide into life again. No doubt it is quite different with their ova.

Mr. J. T. Cunningham contributes an interesting article on Salmonidæ, in which he presents a pretty full synopsis of most of the genera and of all the British species. The life-history of the British forms is given, and some account of the legislation on the subject of our fisheries. The salmon-disease is described. From whence the fresh salmon gets affected would seem to be an, as yet, unsettled problem. Might not one source be frog-spawn? After the tadpoles escape, the gelatinous nidus remaining will sometimes be found permeated with *Saprolegnia ferox*, with ripe oosporangia.

The next biological article of importance in the volume is on the Schizomycetes, by Prof. Marshall Ward, including within this term all those Schizophyta devoid of chlorophyll. The history of these forms, though dating only from 1860, has of late years made rapid progress, and the epitome of this history as here given is full of interest. Most judiciously, while selecting the facts from writings of scientific worth, the author ignores a lot of the rubbish that has appeared in print on the subject. The section on the morphology is very ably written, and the illustrative figures are excellent. Very thoroughly do we agree with the author, that to deny the existence of species in this group is to deny the existence of species altogether. No doubt, before they can be properly defined, the whole life-history of any one of these forms must be known ; and equally certain is it that immense advance in a knowledge of the life-history of many of them has been made since the date of Cohn's brilliant researches. As to the important question, Are the Schizomycetes accompaniments only of disease, or have they any causal relation to the diseased condition? no decided answer is given, the discussion as to details being still an active one. The theory that, by the growth and development of certain forms under certain conditions, the medium in which these forms live may be so atomically altered that new and deadly ptomaines may arise, is not alluded to. To the references to authorities given it may be useful to add Just's *Botanischer Jahresbericht*, which year by year laboriously works out the immense literature of this subject.

Dr. Günther's article on Sharks is well illustrated, and well up to date. There is a capital figure of that most interesting and novel form, *Chlamydoselachus anguineus*, S. Garman, from Japan, of which but two specimens are known, one of these being in the British Museum. In reference to the economic use of these fish, allusion is made to the oil abstracted from the larger forms. We may mention that the oil from the basking shark is of considerable commercial value, and that in the case of the immense *Rhinodon typicus*, which abounds during certain seasons in the seas around the Seychelles Islands, the oil is of excellent quality, and there is little doubt would pay well for collecting, although owing to the differences in the respiratory functions, the difficulty in capturing large fishes is vastly greater than in capturing big mammals.

When we turn from the biological to the mathematical and physical sciences, we do not find such a great wealth of articles, but on the other hand some of them are of the highest order, the only fault about them being, perhaps, their shortness. Such are the articles on "Quaternions" and "Radiation," by Prof. Tait. Prof. Dittmar writes on sea-water, and Prof. Ewing on seismometers. Mr. Herbert Rix, the Assistant Secretary of the Royal Society, gives an account of the history and doings of it, which will be read with interest by many. The chemists have their fair share of interest in the volumes ; prussic acid, pyrotechny, salts, being among the subjects treated of in all their aspects.

Midway between pure science and its applications, we find an important article on screws by Prof. Roland, while in the various applications of science there is a great wealth of admirable articles: railways, river engineering, roads and streets, shipbuilding, public health, are among the subjects of this nature treated of; and always, so far as we can judge, by the best man.

Of the contributions relating to geography, ethnography, and statistics in these volumes, one of the most important is Prince Krapotkine's part of the article "Russia," in which he presents a very lucid account of the leading facts connected with his subject. Like all other races, the Russians are, of course, to some extent a mixed race. In the course of their history they have taken in and assimilated a variety of Finnish and Turco-Finnish elements. The author, however, points out that, notwithstanding this process, the Slavonian type has maintained itself with remarkable persistency, Slavonian skulls ten and thirteen centuries old exhibiting the same anthropological features as are seen in those of our own day. This he accounts for chiefly by the fact that the Slavonians, down to a very late period, maintained gentile organisation and gentile marriage. Dealing with the circumstances of Russia at the present day, Prince Krapotkine says that much still remains to be done for the diffusion of the first elements of a sound education throughout the Empire, and that the endeavours of private persons in this field, and of the *zemstvos*, are for political reasons discouraged by the Government. The Government also does what it can to check the movement in favour of secondary schools where instruction would be based on the study of the natural sciences. It prefers classical gymnasiums. As every one knows, the natural sciences are much cultivated in Russia; and now the scientific societies of old and recognised standing have to compete with a group of new societies which have sprung up in connection with the Universities.

The geography and statistics of Prussia are dealt with by Mr. J. F. Muirhead. Although somewhat hampered by the fact that the physical features of Prussia had already been fully described under "Germany," Mr. Muirhead has brought together much valuable and interesting information both about the country and about the Prussian people. He has, of course, a good deal to say about the flourishing condition of education in Prussia. Of the recruits levied to serve in the army in 1882–83 the proportion of men unable to read or write was only 2 per cent., the rate varying from 9·75 per cent. in Posen to 0·03 in Schleswig-Holstein, where there was only one illiterate recruit among 3662. Mr. Muirhead contributes several other geographical articles—among them, the one on the Rhine, of which he thinks that probably the Tiber alone is of equal historical interest among European rivers. After a full account of the physical aspects of the river and of its relations to industry and trade, he shows how its whole valley was probably occupied at one time by Celtic tribes, and how they were gradually displaced by the advancing Teutons.

The topography and archæology of Rome have been intrusted to Mr. J. H. Middleton, whose thorough knowledge of his subject has enabled him to make the most of the limited space at his disposal. The article is devoted mainly to those buildings of which some remains still exist. The plan of the Forum and nearly all the cuts were measured and drawn by the author specially to illustrate this article.

Mr. George G. Chisholm gives a clear description of the physical features, with an adequate account of the agriculture, mineral wealth, and trade, of Roumania and Servia. He has also a good article on Sardinia. There is an excellent article on the St. Lawrence, by Sir Charles A. Hartley, who points out that the great prosperity and growth of Canada are owing to its unrivalled system of intercommunication, by canal and river, with the vast territories through which the St. Lawrence finds its way from the far-off regions of the Minnesota to the seaboard The statistics of Scotland have been carefully done by. Mr. T. F. Henderson, but the scientific part of the article "Scotland" is remarkable chiefly for Dr. Archibald Geikie's masterly sketch of the physical features of the country and his summary of the facts relating to its geological formations. The article on Siam, by Mr. Coutts Trotter, contains all the information that ordinary readers are likely to want about the physical characteristics and resources of the country, and about Siamese law, education, religion, and art.

It will be evident from what we have said, that although the "Encyclopædia" has already reached its twenty-first volume, there is no falling off either in the care or in the zeal of the editors. If all goes well, it is expected that the whole work will be completed in four more volumes; and we may certainly say that the work has been conducted in such an admirable manner that science will be a great gainer by it, and that it is a production of which everybody concerned may be justly proud.

A TREATISE ON CHEMISTRY

A Treatise on Chemistry. By Sir H. E. Roscoe, F.R.S., and C. Schorlemmer, F.R.S., Professors of Chemistry in the Victoria University, Owens College, Manchester. Vol. III. "The Chemistry of the Hydrocarbons and their Derivatives, or, Organic Chemistry." Part III. (London and New York; Macmillan and Co., 1886.)

IN the present instalment of the organic section of this valuable work the authors begin the consideration of the so-called aromatic compounds—the members of the benzene series.

The attention of chemists had long been directed to a group of organic compounds remarkable for their richness in carbon, and apparently unconnected in any way with the ordinary "fatty compounds"—richer in hydrogen, and correspondingly poorer in carbon—with which organic chemistry chiefly busied itself. Many of the compounds of this anomalous class occurred in Nature as odoriferous principles; this physical property was made the basis of a rough classification, and the name "aromatic compounds," originally employed in its strict sense, was extended so as to embrace the whole class, thus including compounds destitute of aroma. Much was known, in a more or less disjointed fashion, concerning these aromatic compounds; but no attempt had been made to solve the problem of their constitution until Kekulé, in 1865, proposed his well-known benzene formula. This formula has at no time, since it was first introduced, met with universal acceptance; and although, at the present day,